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601. A cubic equation is given as

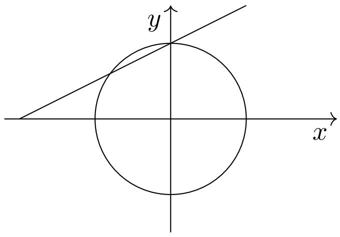
$$(x+2)^3 - (x-2)^3 = 0.$$

Show that this equation has no real roots.

602. Two dice are rolled, and the values are added. By drawing the possibility space, show that 7 is the most likely score.

603. Solve $\dfrac{\sqrt{x}-1}{\sqrt{x}+1} = 2 - \sqrt{x}$.

604. The unit circle $x^2 + y^2 = 1$ and the line segment $\mathbf{r} = t\,\mathbf{i} + \left(1 + \frac{1}{2}t\right)\mathbf{j}$, for $t \in [-2, 1]$ are drawn.



Show that $4/15$ of the length of the line segment lies within the circle.

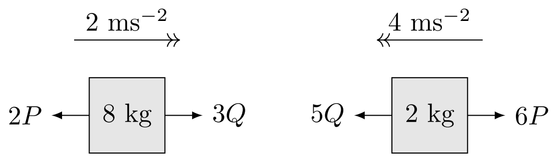
605. Write 16^t in terms of $y = 8^t$.

606. Equations in x and y are given as

$$x^2 + y^2 = 1$$
$$x^2 - y^2 = 2.$$

Show that these have no simultaneous solutions.

607. Two objects are modelled with the following force diagrams. The forces have units of Newtons.



Determine the value of P and of Q.

608. Write down the largest real domains over which the following functions may be defined:

(a) $x \mapsto \sqrt{1-x}$,
(b) $x \mapsto \sqrt{1-x^2}$,
(c) $x \mapsto \sqrt{1-x^3}$.

609. Prove that $\tan\theta + \cot\theta \equiv \operatorname{cosec}\theta \sec\theta$.

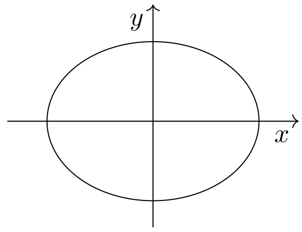
610. Find, in terms of the positive constants a, b, the area of the region of the (x, y) plane whose points simultaneously satisfy the following inequalities:

$$-a < x < a,$$
$$-b < y < b.$$

611. The ellipse shown is given by parametric equations

$$x = 4\cos t,$$
$$y = 3\sin t,$$

where t takes all values in $[0, 2\pi)$.



Write down the greatest and smallest values of the diameter of the ellipse.

612. For some $k \in \mathbb{N}$, the sum of the first k integers is equal to $3k$. Find k.

613. Explain why the factor theorem cannot be used, over the real numbers, to establish whether the expression $(x^2 + 1)$ is a factor of $4x^5 + x + 1$.

614. Two ships leave a port at the same time. Ship A travels on bearing 020° at 14 mph; ship B travels on bearing 280° at 16 mph. To the nearest minute, determine the time taken for the ships to separate by 10 miles.

615. Write the set $\{x \in \mathbb{R} : x^2 - x \leq 0\}$ in interval set notation.

616. Provide a counterexample to the following claim: "If a_n and b_n are APs, then so is $a_n b_n$."

617. For most values of the constants a, b, the following equation has exactly two real roots $x = \pm a$:

$$\frac{(x+a)(x-a)}{(x+b)(x-b)} = 0.$$

Explain the two ways in which the equation could have fewer than two real roots, giving conditions on the constants a and b.

618. Determine all possible values of $\mathbb{P}(A \cap B)$, given that $\mathbb{P}(A) = \frac{1}{2}$ and $\mathbb{P}(B) = \frac{2}{3}$.

619. Factorise $x^3 - 2x^2 - 5x + 6$.

620. It is given that $\frac{d}{dx}\,\mathrm{F}(x) = \mathrm{f}(x)$, and that $\mathrm{F}(0) = 2$, $\mathrm{F}(4) = 6$. Evaluate the following:

(a) $\displaystyle\int_0^4 \mathrm{f}(x)\,dx$,

(b) $\displaystyle\int_0^4 5\,\mathrm{f}(x) - 1\,dx$.

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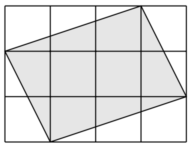
621. The grid shown below consists of unit squares.



(a) Find the area of the shaded parallelogram.

(b) Determine the exact area of the smallest circle into which the shaded parallelogram will fit.

622. State that the following holds, or explain why not: "If there is no resultant force on an object, then it is in equilibrium."

623. An iteration is defined as

$$x_{n+1} = x_n^2 - \frac{3}{x_n + 1}.$$

Show that, if the iteration has a fixed point α, then $(x - \alpha)$ is a factor of the cubic expression $x^3 - x - 3$.

624. Make x the subject of $y = \dfrac{1 + \frac{1}{x+1}}{1 - \frac{1}{x+1}}$.

625. Show that $\displaystyle\int_1^2 \frac{16x + 48}{x^3}\, dx = 26$.

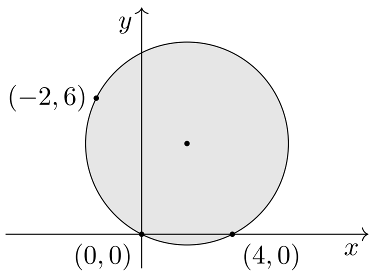
626. A function has instruction $\mathrm{f}(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

(a) Find the largest real domain over which f may be defined.

(b) Without using any calculus, write down the least possible value of $\mathrm{f}(x)$ on this domain.

(c) Hence, state the range of f, giving your answer in set notation.

627. The circle shown passes through the points $(0, 0)$, $(4, 0)$ and $(-2, 6)$.



(a) Explain how you know that the x coordinate of the centre of the circle is 2.

(b) Let the centre be $(2, y)$ and the radius r. Show, using Pythagoras, that r^2 may be expressed as $4 + y^2$ and also as $16 + (6 - y)^2$.

(c) Hence, find the area of the circle.

628. By taking out a common factor, solve the equation

$$(x + 1)^3 - 4x^2 - 4x = 0.$$

629. A particular quadratic function g has $\mathrm{g}(0) = 2$, $\mathrm{g}'(0) = 0$, $\mathrm{g}''(0) = -2$. Sketch the graph $y = \mathrm{g}(x)$, labelling all axis intercepts.

630. Using a Venn Diagram, or otherwise, simplify

(a) $(A' \cap B')'$,

(b) $(A \cup B) \setminus (A' \cap B)$.

631. Simplify $[-1, 0] \cap \left\{x : x^2 < \frac{1}{4}\right\}$.

632. The definite integral below gives the displacement, over a particular time period, for an object moving with constant acceleration:

$$s = \int_{T=0}^{T=t} u + aT\, dT.$$

Carry out the definite integral to prove one of the constant acceleration formulae.

633. Solve $2^x - 2^{1-x} = 1$.

634. On the same axes, for positive constants a, b, m, sketch the graphs

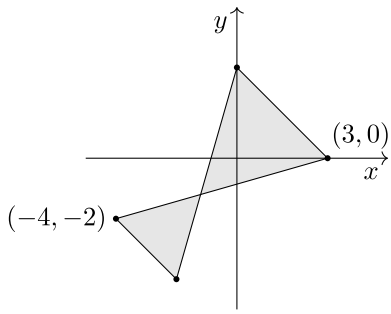
① $\dfrac{y - b}{x - a} = m$, ② $\dfrac{y - b}{x - a - 1} = m$.

635. The derivative of a function f is constant. Show that $\mathrm{f}(x + 1) - \mathrm{f}(x - 1)$ is constant.

636. By considering intersections with the line $y = x$, show that the outputs of $\mathrm{g}(x) = x^2 - 2x + 3$ are always greater than its inputs.

637. Factorise $680x^2 - 842x - 1207$.

638. The shaded region shown has the line $y = x$ as a line of symmetry.



Find the area shaded.

639. The quartic approximation to $\tan^2 x$, for x defined in radians, is $\tan^2 x \approx x^2 + \frac{2}{3}x^4$. Determine the percentage error in this approximation at $x = \frac{1}{6}\pi$.

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640. Show that a rectangle with sides 20 and 28 cm will not fit inside a circle of diameter 34 cm.

641. Prove that, if x and y have a linear relationship, and y and z have a linear relationship, then x and z also have a linear relationship.

642. Assuming the formula $\log_a x + \log_a y = \log_a xy$, either prove or disprove the following:

$$\log_a x + \log_a y + \log_a z = \log_a xyz.$$

643. Write $x^2 + x + 6$ as a polynomial in $(x + 1)$.

644. The cubic equation $(x + a)(x^2 + 4x + b) = 0$, for constants $a, b \in \mathbb{R}$ has exactly one real root. Find the set of possible values of b.

645. Evaluate the following sums:

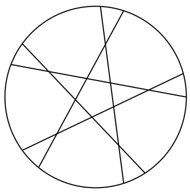
(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...,$

(b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...$

646. Give the acceleration of a lift if accurate weighing scales placed inside it underestimate mass by 15%.

647. When divided by $(x-3)$, the quadratic px^2+x+p leaves no remainder. Find the value of p.

648. The *lazy caterer's sequence* describes the greatest number of pieces into which a circular cake can be cut using n straight line cuts. A set of five such cuts is shown below, providing 16 pieces:



The formula for the lazy caterer's sequence is

$$P_n = \frac{n^2 + n + 2}{2}.$$

Verify this formula for $n = 1, 2, 3, 4$.

649. A straight line L is parallel to the line $2x + 3y = 7$ and passes through the midpoint of $(-10, 0)$ and $(-2, 4)$. Show that L goes through $(6, -6)$.

650. Provide counterexamples to the following:

(a) "All points of inflection are stationary."

(b) "No point of inflection is stationary."

651. In this question, a, b and c are constants. Show that, for $a \neq 0$, the loci of the following equations intersect normally at (b, c):

$$y = a(x - b) + c$$
$$y = -\tfrac{1}{a}(x - b) + c.$$

652. Explain whether it is possible for the two forces of a Newton (NIII) pair to act on the same object.

653. A chemist is measuring acidity during a titration experiment. The pH value k of the contents of a flask is modelled, for the first minute, as having a rate of change, in units of s^{-1},

$$\frac{d}{dt}(k) = 0.32 - 0.012t.$$

(a) State the initial rate of change of pH.

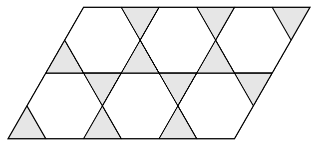
(b) Find the time at which pH begins to fall.

(c) Determine Δk over the first minute.

(d) Explain how you know that pH has no units.

654. Prove that, if the limits of a definite integral are switched, then the value of the integral is negated.

655. A tiling pattern, consisting of regular hexagons and equilateral triangles, is shown in the diagram.



Determine the fraction of the total area that is covered by the triangles.

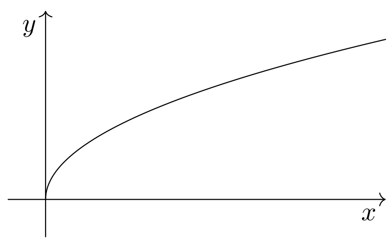
656. A sample is taken, with statistics as follows:

$$n = 25, \quad \sum x = 327, \quad \sum x^2 = 5229.$$

Find the mean and standard deviation.

657. Prove that, if x and y are large positive integers which satisfy *Pell's equation* $x^2 - 2y^2 = 1$, then x/y is an approximation for $\sqrt{2}$.

658. The graph of $y = \sqrt{x}$ is shown below.



Sketch the graph $y = \sqrt{-x}$.

659. Explain the difference in meaning, with regard to changes in t, between the notations Δt, δt, dt.

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660. State, giving a reason, if any of the implications $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$ links the following statements concerning a real number x:

① $x \in \mathbb{Q}'$,

② $x \in \mathbb{R}'$.

661. Determine the fraction of the area of a 2 by 4 cm rectangle which is within 1 cm of a vertex.

662. Solve the equation $|3x - 1| = 2 + |x|$.

663. "The y axis is tangent to $x = y^2 - 8x + 16$." Is this statement true or false?

664. A pair of events X and Y have probabilities, for constants m and n, given as follows:

$$\begin{aligned} \mathbb{P}(X) &= \frac{1}{m}, \\ \mathbb{P}(Y) &= \frac{1}{n}, \\ \mathbb{P}(X \cup Y) &= \frac{m+n-1}{mn}. \end{aligned}$$

Show that X and Y are independent.

665. The largest real domain over which a function g can be defined is $[0, 1]$. Over this domain, g has range $[0, 1]$. State, with a reason, whether each of the following is a well-defined function over the domain $[0, 1]$:

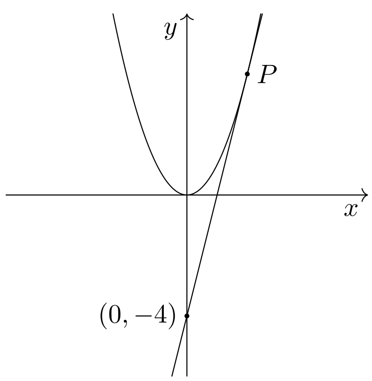
(a) $x \mapsto \mathrm{g}(x) + 2$,

(b) $x \mapsto \mathrm{g}(x + 2)$.

666. A particle is modelled as having initial velocity u and constant acceleration a. Using integration, prove that $s = ut + \frac{1}{2}at^2$.

667. A die has been rolled and the score noted down. Determine whether knowing that the score is even changes the probability that the score is prime.

668. The tangent to the curve $y = x^2$ at point P passes through $(0, -4)$.



(a) Write down the equation of a line which passes through the point $(0, -4)$ with gradient m.

(b) Explain why, if such a line is to be tangent to the curve, then the equation $x^2 - mx + 4 = 0$ must have precisely one root.

(c) Hence, find the coordinates of P.

669. Evaluate $\left[\dfrac{x!}{1 + 2^{-x}}\right]_0^1$.

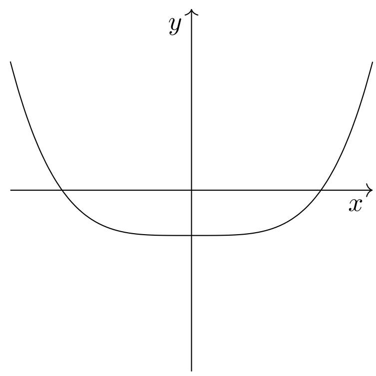
670. The solution of the following equation is the same for all but two values of the constant k.

$$\frac{x^2 - 4}{x^2 - kx} = 0.$$

Write down those values.

671. Find the interquartile range of the standardised normal distribution $N(0, 1)$.

672. Show that $y = (x - 1)^4$ could not possibly be the equation of the following sketched graph:



673. (a) Find $\displaystyle\int_1^N \frac{1}{x^2}\,dx$.

(b) Hence, determine $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$.

674. The first three terms of a geometric sequence are given as a, $a + 2$. $a + 3$. Find a.

675. A car is turning around a corner. Explain how you know that the reaction forces exerted by the road on the wheels and the frictional forces exerted by the road on the wheels are perpendicular.

676. You are given that "the rate of change of y with respect to x is proportional to x^2."

(a) Express the above statement as an equation, using a constant of proportionality k.

(b) You are also told that $\left.\dfrac{dy}{dx}\right|_{x=\sqrt{3}} = 6$. Find k.

(c) By integrating, write y in terms of x.

(d) When $x = 0, y = 6$. Find y when $x = 3$.

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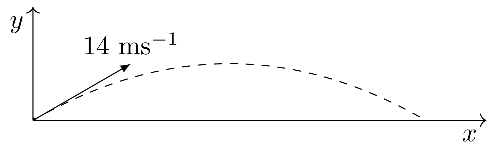
677. A projectile is launched from ground level at speed 14 ms^{-1}, at an angle of 30° above the horizontal.



(a) State the assumptions of the projectile model.

(b) Find the horizontal and vertical components of the initial velocity.

(c) By considering the vertical motion, find the time taken for the projectile to land again.

(d) Hence, show that the range is $10\sqrt{3}$ m.

678. It is given that the quadratic $px^2 + qx + r = 0$ has exactly two roots. Show that $px^2 - qx + r = 0$ also has exactly two roots.

679. Without using calculus, show that $y = 3x + 2$ is tangent to $y = x^3$.

680. Show that the equation of the locus of points equidistant from $y = x$ and $y = -x$ is $xy = 0$.

681. Using a Venn diagram, or otherwise, prove that, for any events A and B,

$$\mathbb{P}(A \cup B) + \mathbb{P}(A \cap B) \equiv \mathbb{P}(A) + \mathbb{P}(B).$$

682. By setting $x = \cos y$, show that

$$\sin(\arccos x) \equiv \sqrt{1 - x^2}.$$

683. The number of people x living in each flat in a particular block is given as follows:

x	1	2	3	4	5	6	7
f	12	22	14	17	15	8	3

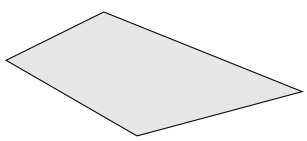
Use a calculator to find the value of

(a) $S_{xx} = \sum (x_i - \bar{x})^2$,

(b) the variance.

684. Find the range of $\text{f}(x) = x^2 - 1$ over $[2, 3]$.

685. The diagram shows quadrilateral Q:



A student rotates Q by 30° anticlockwise. She then describes the resulting shape as $ABCD$, where A is $(0, 0)$, B is $(5, 0)$, C is $(2, 3)$ and D is $(9, 4)$. There is no error in her calculations. Explain the error in her use of notation.

686. By factorising, evaluate $\displaystyle\lim_{k\to 2} \frac{4k^2 - 16}{k^2 - 2k}$.

687. Two astronauts, each of mass m, are rotating around each other in space, holding onto the ends of a light cord of length d. The tension in the cord is the only force acting on them. Each astronaut is moving in a circle at speed v.

At time $t = 0$, one of them lets go. Show that the distance between them in the subsequent motion is given by $x = \sqrt{4v^2t^2 + d^2}$.

688. Describe all functions f for which f′ is constant.

689. Using a double-angle identity, it can be shown that

$$2 \sin 18° = 1 - 4 \sin^2 18°.$$

Use this to determine the exact value of $\sin 18°$.

690. A straight line is given parametrically as $x = 1 - \lambda$, $y = 3 + 2\lambda$, for $\lambda \in \mathbb{R}$. This line is then translated by the vector $2\mathbf{i} + 3\mathbf{j}$. Write down the equation of the new line, in the same form.

691. The interior angles of a triangle are in AP. Show that one of the angles must be $\frac{\pi}{3}$ radians.

692. Find the set of values of x for which the derivative of $y = x^2 - 10x + 2$ is non-negative.

693. A function taking real inputs has instruction

$$\text{g} : x \mapsto \frac{1}{\sqrt{x^2 + px + q}}.$$

You are given that the largest domain over which this function is well defined is $(\infty, -3) \cup (4, \infty)$. Determine the value of p and the value of q.

694. It is given that $x + y$ is constant. Find $\dfrac{dy}{dx}$.

695. State whether or not the line $x - 2y = 0$ intersects the following graphs:

(a) $y = |x| + 1$,

(b) $y = |x| - 1$,

(c) $x = |y| + 1$,

(d) $x = |y| - 1$.

696. The *arithmetic mean* and *geometric mean* of +ve numbers x, y are defined as $\frac{1}{2}(x + y)$ and $\sqrt{xy}$.

You are given that the difference between positive numbers p and q is six fifths of their arithmetic mean. Show that the difference is three halves of their geometric mean.

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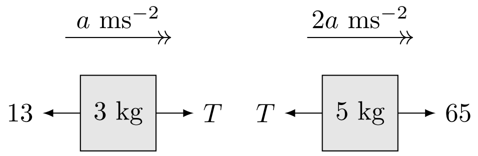
697. Describe formally the transformation which takes the graph $y = x^2$ onto the graph $y = (x-3)^2$.

698. Two objects are modelled as depicted:



(a) Find a and T.

(b) The forces with magnitude T are a Newton III pair of tensions exerted via a string. Explain how you know that the string is extensible.

699. Evaluate $1024 + 512 + 256 + 128 + \ldots$.

700. A square is drawn inside $x^2 + 4x + y^2 - 6y = 0$, with its four vertices on the circumference.

(a) Complete the square in x and y, and hence find the radius of the circle.

(b) Show that the square has area 26.

——— End of 7th Hundred ———

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